

Indian Institute of Information Technology Allahabad
Mid-sem Question paper (Tentative Marking Scheme)

Course Name: Real Analysis
Course Code: DA221
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Program Name: B.Tech. (3rd SEM)
MM: 25

Important Instructions:

1. All notations are standard, and the same as used in classes and lecture notes.
 2. Use of any electronic gadgets including smart watches is not allowed.
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1. Check whether the following statements are true. Give a proper justification. [10 = 2 × 5]

- (a) If a series $\sum a_n$ of real numbers is convergent, then $a_n \rightarrow 0$.

Solution True.

Note that $a_n = S_n - S_{n-1}$. Suppose $\sum a_n$ is convergent and converges to l . Then $\lim a_n = \lim S_n - \lim S_{n-1} = l - l = 0$.

- (b) Let (x_n) be a sequence. Then $x_n \rightarrow l$ if and only if $|x_n| \rightarrow |l|$.

Solution False.

Let (x_n) , where $x_n = (-1)^n$. Then $|x_n| \rightarrow 1$ but x_n is not convergent.

- (c) $\sum_{n=2}^{\infty} \frac{1}{n^2 \log n}$ is divergent.

Solution False.

Let $a_n = \frac{1}{n^2 \log n}$ and $b_n = \frac{1}{n^2}$. Then $\lim \frac{a_n}{b_n} = 0$. Since $\sum b_n$ is convergent, by limit comparison test, the series $\sum a_n = \sum_{n=2}^{\infty} \frac{1}{n^2 \log n}$ is also convergent.

- (d) Let $\sum a_n$ be a convergent series and (b_n) be a bounded sequence. Then $\sum a_n b_n$ is always convergent.

Solution False.

Let $a_n = \frac{(-1)^n}{n}$ and $b_n = (-1)^n$. Then $\sum a_n b_n = \sum \frac{1}{n}$ is divergent.

- (e) If a function f is continuous on an interval (a, b) , then it is bounded.

Solution False.

Consider the continuous function $f(x) = \frac{1}{x}$ on $(0, 1)$. Then $f(x) \rightarrow \infty$ as $x \rightarrow 0$.

2. Find the supremum and infimum of the set $S = \{\frac{m}{m+n} : m, n \in \mathbb{N}\}$. [4]

Solution. Observe that $0 < l = \frac{m}{m+n} < 1$ for every $l \in S$.

Claim that $\inf(S) = 0$. [1]

On the contrary, suppose $x_0 > 0$ is the infimum of the set S .

Then by Archimedean property for $x = x_0 > 0$ and $y = 1$, $\exists N \in \mathbb{N}$, say $N = n + 1$, such that $(n + 1)x_0 > 1$, which implies, $x_0 > \frac{1}{n+1} \in S$. This contradicts the fact that $\inf(S) = x_0$. [1]

Claim that $\sup(S) = 1$. [1]

On the contrary, suppose $y_0 < 1$ is the supremum of the set S .

Then by Archimedean property, for $x = 1 - y_0 > 0$ and $y = 1$, $\exists N \in \mathbb{N}$, say $N = n + 1$. such that $(n + 1)(1 - y_0) > 1$, which implies $y_0 < \frac{n}{n+1} \in S$. A contradiction. [1]

3. Let $x_n \geq 0$ for every $n \in \mathbb{N}$. Suppose (x_n) is a convergent sequence such that $x_n \rightarrow l$. Show that $l \geq 0$. [3]

Solution. Suppose $l < 0$. Choose $\epsilon_0 = -l > 0$. [1]

Then for the $\epsilon_0 > 0$ there exists $N \in \mathbb{N}$ such that $|x_n - l| < -l$. [1]

$\implies x_n < 0$ for all $n \geq N$. A contradiction. [1]

4. Show that the equation $x^{13} + 7x^3 - 5 = 0$ has exactly one real roots. [3]

Solution. Let $f(x) = x^{13} + 7x^3 - 5$. Then $f(0) = -5 < 0$ and $f(1) = 3 > 0$. So by Intermediate value theorem $\exists x_0 \in (0, 1)$ such that $f(x_0) = 0$. [1]

Since $f(x) < 0$ when $x < 0$, so if $f(x)$ has another root say x_1 then $x_1 > 0$. [1]

Now $f(x_0) = f(x_1) = 0$, so by Rolles's theorem $\exists c > 0$ between x_0 and x_1 such that $f'(c) = 13c^{12} + 21c^2 = 0$. Which is not possible. [1]

5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfy $f(x + y) = f(x) + f(y) \forall x, y \in \mathbb{R}$. If f is continuous at 0, show that f is continuous at every point $c \in \mathbb{R}$. [5]

Solution. Note that $f(0) = 0$, $f(-x) = -f(x)$, $f(x - y) = f(x) - f(y)$. [1+1+1]

Now let $x_0 \in \mathbb{R}$ and $x_n \rightarrow x_0$. Then $f(x_n) - f(x_0) = f(x_n - x_0)$. Since f is continuous at 0 so $f(x_n - x_0) \rightarrow f(0) = 0$ as $x_n - x_0 \rightarrow 0$. That is $f(x_n) \rightarrow f(x_0)$ as $x_n \rightarrow x_0$. [2]